This is an equality of sets, so $\langle \text{acronymref} | \text{definition} | \text{SE} \rangle$ applies.The "easy" half first. Show that $X = \langle \text{spn} | \times \rangle \subseteq \langle \text{spn} | \times \rangle = Y$.
Choose $\langle \text{vect} | x \rangle \in X$. Then $\langle \text{vect} | x \rangle = a_1(\langle \text{vect} | v \rangle_1 + \langle \text{vect} | v \rangle_2) + a_2(\langle \text{vect} | v \rangle_1 - \langle \text{vect} | v \rangle_2)$ for some scalars a_1 and a_2. Then,

$$\begin{aligned}
\langle \text{error} | \text{compound vect} \rangle &= a_1(\langle \text{vect} | v \rangle_1 + \langle \text{vect} | v \rangle_2) + a_2(\langle \text{vect} | v \rangle_1 - \langle \text{vect} | v \rangle_2) \\
&= a_1 \langle \text{vect} | v \rangle_1 + a_1 \langle \text{vect} | v \rangle_2 + a_2 \langle \text{vect} | v \rangle_1 + (-a_2) \langle \text{vect} | v \rangle_2 \\
&= (a_1 + a_2) \langle \text{vect} | v \rangle_1 + (a_1 - a_2) \langle \text{vect} | v \rangle_2
\end{aligned}$$

`Esta es una igualdad de conjuntos, asi qeu aplicamos la defenicion SE [682]. La primera mitad muestra que:`

$X = (\{\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2\}) \subseteq (\{\vec{v}_1, \vec{v}_2\}) = Y$

`Escoja un` $x \in X$`.Entonces` $x = a_1(\vec{v}_! + \vec{v}_2) + a_2(\vec{v}_1 - \vec{v}_2)$ `para cualquier escalares` a_1 `y` a_2.

`Entonces:`

$x = a_1(v_1 + v_2) + a_2(v_1 - v_2)$

$= a_1 v_1 + a_1 v_2) + a_2 v_1 - a_2 v_2)$

$= (a_1 + a_2) v_1 + (a_1 - a_2) v_2$

which qualifies $\langle \text{vect} | x \rangle$ for membership in Y, as it is a linear combination of $\langle \text{vect} | v \rangle_1$, $\langle \text{vect} | v \rangle_2$.Now show the opposite inclusion, $Y = \langle \text{spn} | \times \rangle \subseteq \langle \text{spn} | \times \rangle = X$.
Choose $\langle \text{vect} | y \rangle \in Y$. Then there are scalars b_1, b_2 such that $\langle \text{vect} | y \rangle = b_1 \langle \text{vect} | v \rangle_1 + b_2 \langle \text{vect} | v \rangle_2$. Rearranging, we obtain,

$$\begin{aligned}
\langle \text{error} | \text{compound vect} \rangle &= b_1 \langle \text{vect} | v \rangle_1 + b_2 \langle \text{vect} | v \rangle_2 \\
&= \frac{b_1}{2}[(\langle \text{vect} | v \rangle_1 + \langle \text{vect} | v \rangle_2) + (\langle \text{vect} | v \rangle_1 - \langle \text{vect} | v \rangle_2)] + \frac{b_2}{2}[(\langle \text{vect} | v \rangle_1 + \langle \text{vect} | v \rangle_2) - (\langle \text{vect} | v \rangle_1 - \langle \text{vect} | v \rangle_2)] \\
&= \frac{b_1 + b_2}{2}(\langle \text{vect} | v \rangle_1 + \langle \text{vect} | v \rangle_2) + \frac{b_1 - b_2}{2}(\langle \text{vect} | v \rangle_1 - \langle \text{vect} | v \rangle_2)
\end{aligned}$$

`X cumple con los requisitos para pertenecer a Y, ya que es combinacion lineal de`$\vec{v}_1, \vec{v}_2$`. Ahora muestre la inclusion opuesta:`$Y = (\{\vec{v}_1, \vec{v}_2\}) \subseteq (\{\vec{v}_1 + \vec{v}_2, \vec{v}_1 - \vec{v}_2\}) = X$.

`Escoja un` $\vec{y} \subseteq Y$`. Entonces hay escalares` b_1, b_2 `tales que:` $\vec{y} = b_1 \vec{v}_1 + b_2 \vec{v}_2$. `Reorganizando tenemos:`

$\vec{y} = b_1 \vec{v}_1 + b_2 \vec{v}_2$

$= \frac{b_1}{2}[(v_1 + v_2) + (v_1 - v_2)] + \frac{b_2}{2}[(v_1 + v_2) - (v_1 - v_2)]$

$= \frac{b_1 + b_2}{2}(v_1 + v_2) + \frac{b_1 - b_2}{2}(v_1 - v_2)$

This is an expression for $\langle \text{vect} | y \rangle$ as a linear combination of $\langle \text{vect} | v \rangle_1 + \langle \text{vect} | v \rangle_2$ and $\langle \text{vect} | v \rangle_1 - \langle \text{vect} | v \rangle_2$, earning $\langle \text{vect} | y \rangle$ membership in X. Since X is a subset of Y, and vice versa, we see that $X = Y$, as desired.

`Este es el vector` $\vec{y}$ `expresado como combinacion lineal de` $v_1 + v_2$ `y` $v_1 - v_2$, `obteniendo` $y \subseteq X$. `Dado que` X `es subconjunto de` Y `y viceversa, vemos que` $X = Y$, `como lo deseado`